

PHYS1112 Midterm Exam

Formulae and Constants

Constants:

Acceleration due to gravity $g = 9.80 \text{ m/s}^2$.

Gravitational constant $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

Avogadro's number $N_A = 6.022 \times 10^{23} \text{ molecules/mol}$

Boltzmann constant $k = 1.38 \times 10^{-23} \text{ J/K}$

Gas constant $R = 8.31 \text{ J/mol} \cdot \text{K}$

Atmospheric pressure, $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

Calories: $1 \text{ cal} = 4.186 \text{ J}$

Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Speed of sound in air at room temperature = 340 m/s

Astronomical Data:

Body	Mass (kg)	Radius (m)	Radius of its orbital motion (m) (center-to-center)
Sun	1.99×10^{30}	6.96×10^8	—
Earth	5.97×10^{24}	6.37×10^6	1.50×10^{11}
Moon	7.35×10^{22}	1.74×10^6	3.84×10^8

Formulae

Constant acceleration motions:

Straight-line motion

$$a_x = \text{constant}$$

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$$

Fixed-axis rotation

$$\alpha_z = \text{constant}$$

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$$

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$$

$$\text{Center of Gravity: } \vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}, \quad y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i}, \quad z_{\text{cm}} = \frac{\sum m_i z_i}{\sum m_i}$$

Rotational speed: $v_{\text{tan}} = \omega r$

Tangential acceleration in rotation: $a_{\text{tan}} = \frac{dv_{\text{tan}}}{dt} = \alpha r$

Radial acceleration in rotation: $a_{\text{rad}} = v^2/r = \omega^2 r$

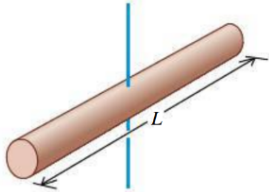
Moment of inertia: $I = \sum m_i r_i^2 \rightarrow \int r^2 dm = \int r^2 \rho dV$

Parallel axis theorem: $I_p = I_{\text{cm}} + Md^2$

Moments of inertia of various bodies:

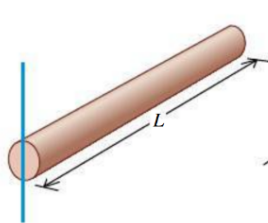
(a) Slender rod,
axis through center

$$I = \frac{1}{12}ML^2$$



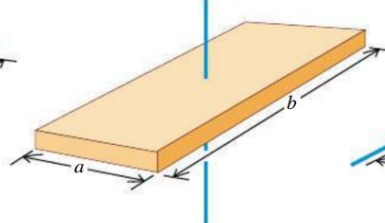
(b) Slender rod,
axis through one end

$$I = \frac{1}{3}ML^2$$



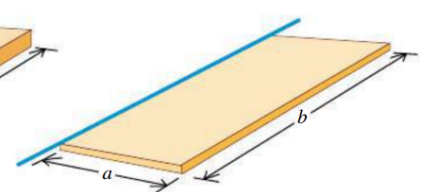
(c) Rectangular plate,
axis through center

$$I = \frac{1}{12}M(a^2 + b^2)$$



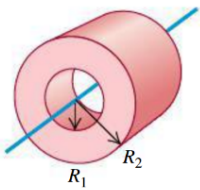
(d) Thin rectangular plate,
axis along edge

$$I = \frac{1}{3}Ma^2$$



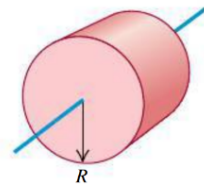
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



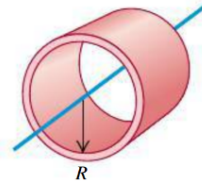
(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$



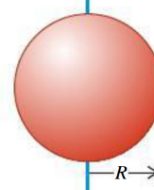
(g) Thin-walled hollow
cylinder

$$I = MR^2$$



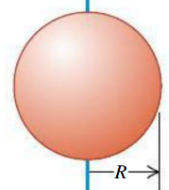
(h) Solid sphere

$$I = \frac{2}{5}MR^2$$



(i) Thin-walled hollow
sphere

$$I = \frac{2}{3}MR^2$$



Trigonometric Relations:

$$\sin A \pm \sin B = 2 \sin \left(\frac{A \pm B}{2} \right) \cos \left(\frac{A \mp B}{2} \right)$$

$$\cos A + \cos B = 2 \cos \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right)$$

$$\cos A - \cos B = -2 \sin \left(\frac{A + B}{2} \right) \sin \left(\frac{A - B}{2} \right)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\sin^2 \frac{A}{2} = \frac{1}{2}(1 - \cos A)$$

$$\cos^2 \frac{A}{2} = \frac{1}{2}(1 + \cos A)$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan^2 \frac{A}{2} = \frac{1 - \cos A}{1 + \cos A}$$

Series Expansion:

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots, \quad |x| < \pi$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, \quad |x| < 1$$

Integrals:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right), \quad \left| \tan^{-1} \left(\frac{x}{a} \right) \right| < \frac{\pi}{2}$$

$$\int \frac{x dx}{a^2 + x^2} = \frac{1}{2} \ln(a^2 + x^2)$$

$$\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2} \ln \left(\frac{x^2}{a^2 + x^2} \right)$$

$$\int \frac{dx}{a^2 x^2 - b^2} = \frac{1}{2ab} \ln \left(\frac{ax - b}{ax + b} \right)$$

$$\int \frac{dx}{\sqrt{a + bx}} = \frac{2}{b} \sqrt{a + bx}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$\int \tan x dx = -\ln |\cos x|$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$$

$$\int x \ln(x) dx = x \ln(x) - x$$